Radiation Fundamentals

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Mechanical Engineering 483

Alternative Energy Engineering II

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Outline

- Electromagnetic radiation
- Black body
  - Stefan-Boltzmann law
  - Proportion of radiation in a waveband
- Directional exchange, radiation intensity and radiosity
- Radiation properties: emissivity, absorbtivity, and reflectivity
  - Wavelength and directional dependence
  - Kirchoff's Law

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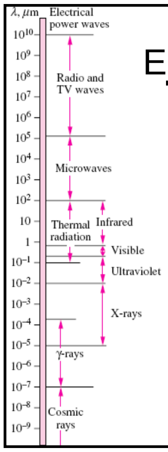

Electromagnetic Radiation

- Radiation heat transfer by electromagnetic radiation
  - Part of much larger spectrum
  - Thermal radiation transfers heat without contact
    - Use of fire or electric resistance heating are best examples
    - Thermal radiation lies in infrared and visible part of spectrum (with some in ultraviolet)

Figure 12-3 from Çengel, *Heat and Mass Transfer* 3

EM Radiation Properties

- Wavelength, λ, ranges from 10^{-9} to 10^{10} μm is distance between wave peaks
- EM waves travel at speed of light = 299,792,458 m/s (in a vacuum)
- Frequency, $\nu = c/\lambda$, units of Hz = s^{-1}
- Radian frequency $\omega = 2\pi\nu$, units s^{-1}
- For ν = 60 Hz = 60 s^{-1}, λ = (299,792,458 m/s) / (60 s^{-1}) $\approx 5\times10^6$ m $= 5\times10^{12}$ μm

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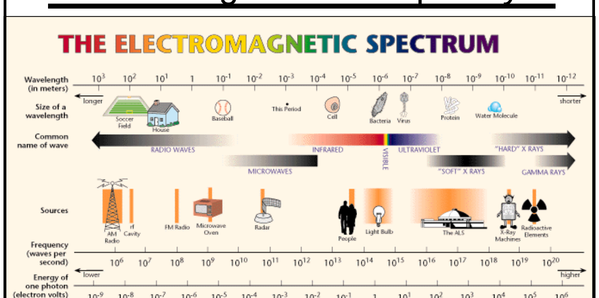
Wavelength and Frequency



http://www.lbl.gov/MicroWorlds/ALSTool/EMSpec/EMSpec2.html

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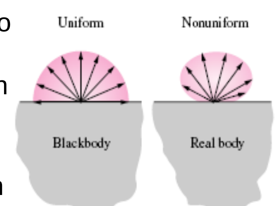
Black-body Radiation

- Perfect emitter – no surface can emit more radiation than a black body
- Diffuse emitter – radiation is uniform in all directions



- Perfect absorber – all radiation striking a black body is absorbed

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Figure 12-7 from Çengel, *Heat and Mass Transfer* 6

Black-Body Radiation II

- Basic black body equation: $E_b = \sigma T^4$
  - E_b is total black-body radiation energy flux W/m^2 or Btu/hr·ft^2
  - σ is the Stefan-Boltzmann constant
    - $\sigma = 5.670\times10^{-8}$ W/m^2·K^4
    - $\sigma = 0.1714\times10^{-8}$ Btu/hr·ft^2·R^4
  - Must use absolute temperature
- Radiation flux varies with wavelength
  - $E_{b\lambda}$ is flux at given wavelength, λ

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Stefan-Boltzmann Constant

- Found experimentally, but later analysis relates σ to other fundamental constants
  - $\sigma = 2\pi^5k^4/(15h^3c^2)$
  - k = Boltzmann's constant = 1.38065×10^{-23} J/K (molecular gas constant) = $R_u/N_{Avagadro}$
  - h = Planck's constant = 6.62607×10^{-34} J·s
    - First notion of quantum mechanics that energy associated with a wave, $\varepsilon = h\nu = hc/\lambda$
  - c = 299,792,458 m/s = speed of light in a vacuum

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Black-body Radiation Spectrum

- Energy (W/m^2) emitted varies with wavelength and temperature
- E_{bl} is spectral radiation
  - Units are W/(m^2·μm)
  - Meaning: fraction of black body radiation in range $\Delta\lambda$ about wavelength λ
- Maximum occurs at λT = 2897.8 μm·K
  - T increase shifts peak shift to lower λ
- Diagram on next chart

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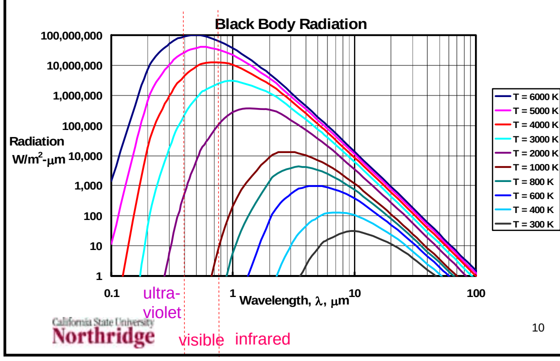
Spectral $E_{b\lambda}$



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Spectral Black-body Energy

- $E_{b\lambda}d\lambda$ = black-body emissive power in a wavelength range $d\lambda$ about λ
  - Typical units for $E_{b\lambda}$ are W/m^2·μm or Btu/hr·ft^2·μm

$$E_{b\lambda}d\lambda = \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)}d\lambda$$

- $C_1 = 2\pi hc^2$ = 3.74177 W·μm^4/m^2
- $C_2 = hc/k$ = 14387.8 μm/K
  - h = Planck's constant, c = speed of light in vacuum, k = Boltzmann's constant

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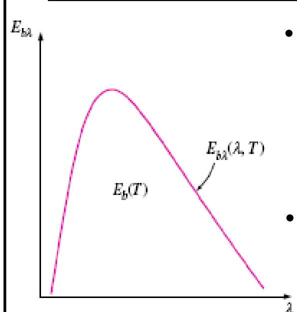
Total Energy



- Total energy is integral over all wavelengths

$$E_b = \int_0^\infty E_{b\lambda}d\lambda = \sigma T^4$$

- Recall that $\sigma = 2\pi^5k^4/(15h^3c^2)$

California State University Northridge Figure 12-11 from Çengel, *Heat and Mass Transfer* 12

Integral Proof

- Show $\int E_{b\lambda} d\lambda = \sigma T^4$ on this and next chart
  - First get common variable $C_2/\lambda T = C_2/y$

$$E_b=\int_{\lambda=0}^{\lambda=\infty}E_{b\lambda}d\lambda=\int_{\lambda=0}^{\lambda=\infty}\frac{C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)}d\lambda=T^4\int_{\lambda T=0}^{\lambda T=\infty}\frac{C_1}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)}d(\lambda T)$$

$$\frac{E_b}{T^4}=\int_{y=0}^{y=\infty}\frac{C_1}{y^5\left(e^{C_2/y}-1\right)}\frac{C_2^5}{C_2^5}dy=\int_{C_2/y=\infty}^{C_2/y=0}\frac{C_1}{y^5\left(e^{C_2/y}-1\right)}\frac{C_2^5}{C_2^5}dy$$

- Define $z = C_2/y$ and get dy in terms of dz

$$z=\frac{C_2}{y}\quad\Rightarrow\quad dz=-\frac{C_2}{y^2}dy=-\frac{C_2}{(C_2/z)^2}dy\quad\Rightarrow\quad dy=-\frac{C_2}{z^2}dz$$

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Integral Proof II

- Get single variable z and integrate

$$\frac{E_b}{T^4}=\int_{C_2/y=\infty}^{C_2/y=0}\frac{C_1}{y^5\left(e^{C_2/y}-1\right)}\frac{C_2^5}{C_2^5}dy=\int_{z=\infty}^{z=0}\frac{C_1z^5}{\left(e^z-1\right)}\frac{1}{C_2^5}dy$$

$$=-\int_{z=\infty}^{z=0}\frac{C_1z^5}{\left(e^z-1\right)}\frac{1}{C_2^5}\frac{C_2}{z^2}dz=\frac{C_1}{C_2^4}\int_{z=0}^{z=\infty}\frac{z^3}{\left(e^z-1\right)}dz=\frac{C_1}{C_2^4}\frac{\pi^4}{15}$$

  - Standard integral found from Matlab command int('z^3/(exp(z)-1)',0,inf)

$$E_b=\frac{C_1}{C_2^4}\frac{\pi^4}{15}T^4=\frac{2\pi hc^2}{(hc/k)^4}\frac{\pi^4}{15}T^4=\frac{2\pi^5k^4}{15h^3c^2}T^4=\sigma T^4$$

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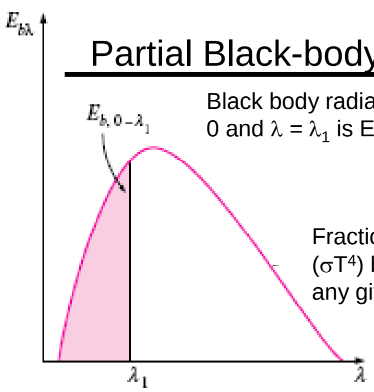
Partial Black-body Power



Black body radiation between λ = 0 and $\lambda = \lambda_1$ is $E_{b,0-\lambda1}$

$$E_{b,0-\lambda_1}=\int_0^{\lambda_1}E_{b\lambda}d\lambda$$

Fraction of total radiation (σT^4) between λ = 0 and any given λ is f_λ

$$f_\lambda=\frac{1}{\sigma T^4}\int_0^{\lambda}E_{b\lambda}d\lambda'$$

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Figure 12-13 from Çengel, *Heat and Mass Transfer*

Radiation Tables

- Can show that f_λ is function of λT

$$f_\lambda=\frac{1}{\sigma T^4}\int_0^{\lambda}E_{b\lambda}d\lambda=\frac{1}{\sigma T^4}\int_0^{\lambda}\frac{C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)}d\lambda=\frac{1}{\sigma}\int_0^{\lambda T}\frac{C_1}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)}d(\lambda T)$$

- Radiation tables give f_λ versus λT
  - See table 12-2, page 118 in Hodge
  - Extract from similar table shown at right

Blackbody radiation functions f_λ

λT, μm · K	f_λ
200	0.000000
400	0.000000
600	0.000000
800	0.000016
1000	0.000321
1200	0.002134
1400	0.007790
1600	0.019718
1800	0.039341
2000	0.066728

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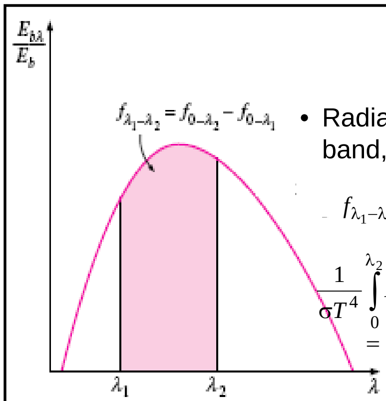
$\Delta\lambda$



- Radiation in finite band, $\Delta\lambda$

$$f_{\lambda_1-\lambda_2}=\frac{1}{\sigma T^4}\int_{\lambda_1}^{\lambda_2}E_{b\lambda}d\lambda=$$

$$\frac{1}{\sigma T^4}\int_0^{\lambda_2}E_{b\lambda}d\lambda-\frac{1}{\sigma T^4}\int_0^{\lambda_1}E_{b\lambda}d\lambda$$

$$=f(\lambda_2T)-f(\lambda_1T)$$

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Figure 12-14 from Çengel, *Heat and Mass Transfer*

Sample Problem

- A conventional light bulb has a filament temperature of 4000ºF. Find the fraction of visible radiation from this filament, if it is a black body.
- **Given:** T = 4000ºF and visible region
- **Find:** Fraction of total radiation in region
- **Missing information:** Visible region is between 0.4 μm and 0.76 μm
- **Conversion:** 4000ºF = 4460 R = 2478 K

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Sample Problem Solution

- Compute λT at λ_1 and λ_2 and find corresponding f_λ values in Table 6.2
  - $\lambda_1 T$ = (0.4 μm)(2478 K) = 991 μm·K
  - $\lambda_1 T$ = (0.79 μm)(2478 K) = 1883 μm·K
  - $f(\lambda_1 T)$ = 0.000289 (interpolation in table)
  - $f(\lambda_2 T)$ = 0.04980 (interpolation in table)
- Fraction in visible range = 0.04980 – 0.000289 = 0.0495 or about 5% in visible range for conventional lighting

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Radiation Exchange

- In general, radiation leaving a surface can vary in direction
  - Ideal of diffuse radiation is uniform in all directions
  - Need coordinate system for radiation leaving a surface
  - Look at hemisphere on top of surface and use spherical coordinate system
  - $I(\theta,\phi)$ is radiation intensity in direction (θ,ϕ)
    - See chart after next for diagram

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Review Spherical Coordinates

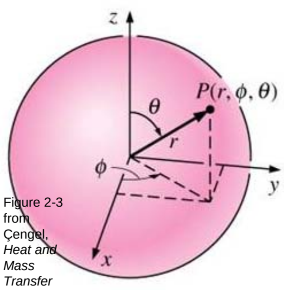

Figure 2-3 from Çengel, *Heat and Mass Transfer*

- Use angular coordinates ϕ and θ
- ϕ is polar angle in x-y plane
- θ is azithmuthal angle with z axis

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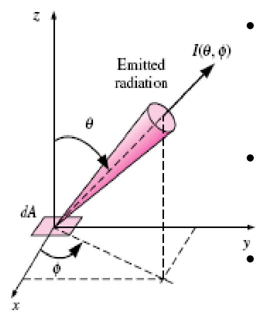
Radiation Intensity

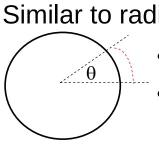


- Radiation intensity, I, is radiation in a particular direction, (θ,ϕ)
- Intensity depends on area at outer surface of cone
- Measure of this surface is solid angle

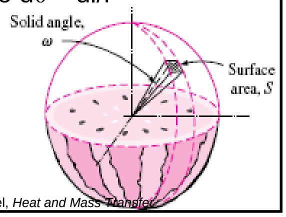
California State University Northridge Figure 12-16 from Çengel, *Heat and Mass Transfer* 22

Solid Angle

- Similar to radian angular measure in 2D



  - Arc length, $l = r\theta$ so $\theta = l/r$
  - Differential arc length $dl = rd\theta$ so $d\theta = dl/r$
- Partial surface area, $S = r^2\omega$ so $\omega = S/r^2$ (Total area = $4\pi r^2$)
- Units for ω are called steradians (sr)



California State University Northridge Figure 12-17 from Çengel, *Heat and Mass Transfer*

$d\omega = \sin\theta \, d\theta d\phi$

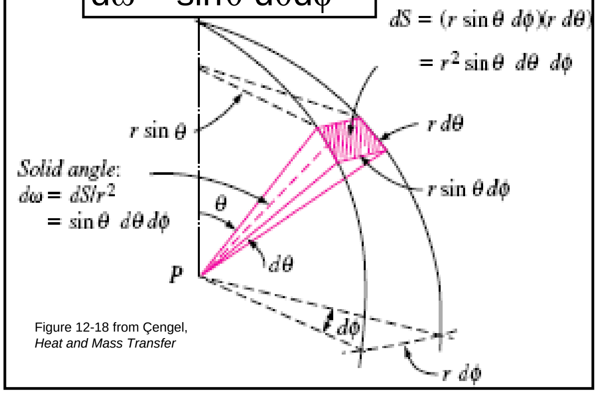
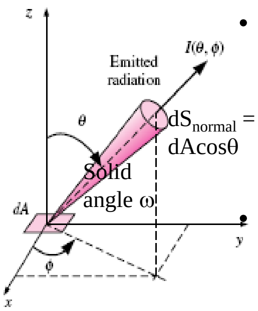


Figure 12-18 from Çengel, *Heat and Mass Transfer*

Radiation Intensity II



- Radiation intensity, I, is radiation energy in a particular direction, (θ,φ) per unit area normal to the direction, per unit solid angle, ω
- Normal area is projection of dA normal to direction = dAcosθ

Figure 12-16 from Çengel, *Heat and Mass Transfer*

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Radiation Intensity III

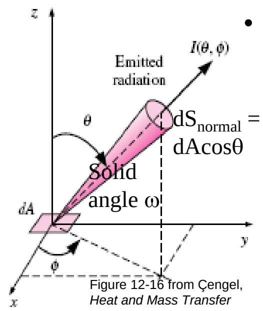

Figure 12-16 from Çengel, *Heat and Mass Transfer*

- Emitted intensity, I_e, is radiation energy, dQ_e, in a particular direction, (θ,φ) per unit area normal to the direction, per unit solid angle, ω

$$I_e(\theta,\phi) = \frac{d\dot{Q}_e}{dA\cos\theta\, d\omega} = \frac{d\dot{Q}_e}{dA\cos\theta\sin\theta\, d\theta d\phi}$$

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For integration over hemisphere: $0 \le \phi \le 2\pi$ and $0 \le \theta \le \pi/2$

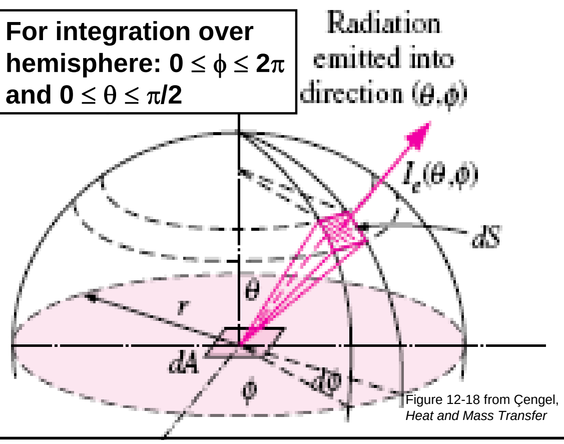

Figure 12-18 from Çengel, *Heat and Mass Transfer*

Emissive Power

- Radiation flux for emitted radiation (energy per unit area of surface)

$$dE = \frac{d\dot{Q}_e}{dA\cos\theta\, d\omega} = I_e(\theta,\phi)\cos\theta\sin\theta\, d\phi$$

$$E = \int_{hemi-sphere} dE = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I_e(\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi$$

For constant I_e, $E = \pi I_e$

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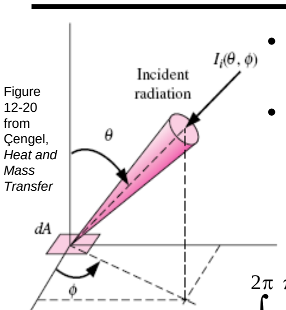
Irradiation G

Figure 12-20 from Çengel, *Heat and Mass Transfer*



- I_i = incident intensity
  – function of direction
- G = total radiation impinging on surface

$$G = \int_{hemi-sphere} dG = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I_i(\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi$$

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Radiosity, J, total intensity leaving surface (sum of directly emitted plus reflected)

J = $I_{emitted}$ + $I_{reflected}$

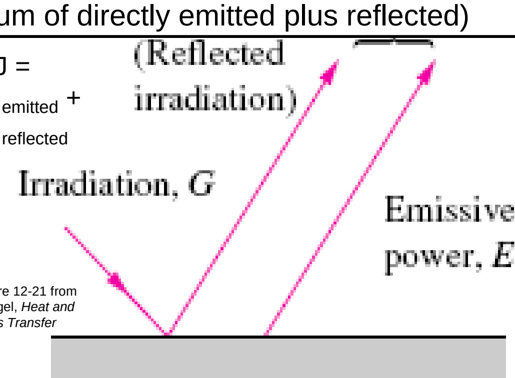

Figure 12-21 from Çengel, *Heat and Mass Transfer*

Spectral Quantities

- Previous discussions of I, E, G, and J have not considered wavelength
- Can define $I_{e,\lambda}$, $I_{i,\lambda}$, and $I_{e+r,\lambda}$
  - Called "spectral" quantities
- Previous quantities are then integrals over all wavelengths

$$I_e = \int_0^\infty I_{e,\lambda} d\lambda \qquad I_i = \int_0^\infty I_{i,\lambda} d\lambda \qquad I_{e+r} = \int_0^\infty I_{e+r,\lambda} d\lambda$$

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Emissivity

- Emissivity, ε, is ratio of actual emissive power to black body emissive power
  - May be defined on a directional and wavelength basis, $\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) = I_{\lambda,e}(\lambda,\theta,\phi,T)/I_{b\lambda}(\lambda,T)$, called spectral, directional emissivity
  - Total directional emissivity, average over all wavelengths, $\varepsilon_\theta(\theta,\phi,T) = I_e(\theta,\phi,T)/I_b(T)$
  - Spectral hemispherical emissivity average over directions, $\varepsilon_\lambda(\lambda,T) = I_\lambda(\lambda,T)/I_{b\lambda}(\lambda,T)$
  - Total hemispheric emissivity = $E(T)/E_b(T)$

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Emissivity Assumptions

- Diffuse surface – emissivity does not depend on direction
- Gray surface – emissivity does not depend on wavelength
- Gray, diffuse surface – emissivity is the does not depend on direction or wavelength
  - Simplest surface to handle and often used in radiation calculations

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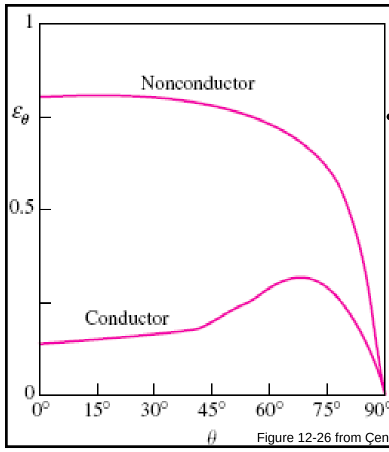

ε_θ

- In general, total directional emissivity is constant over most of surface

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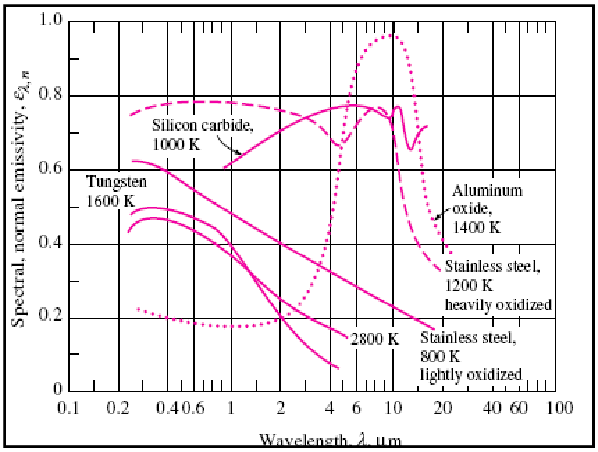
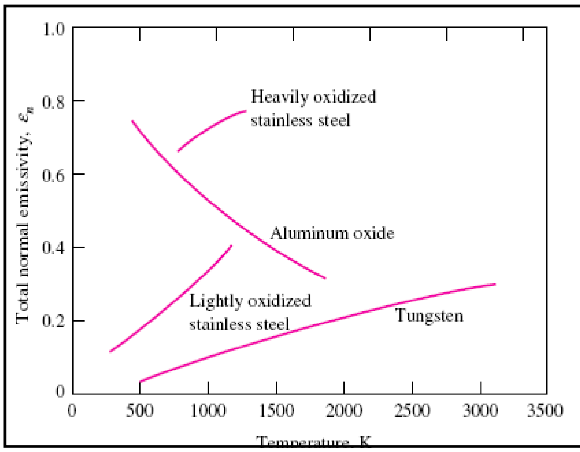
Figure 12-26 from Çengel, *Heat and Mass Transfer*





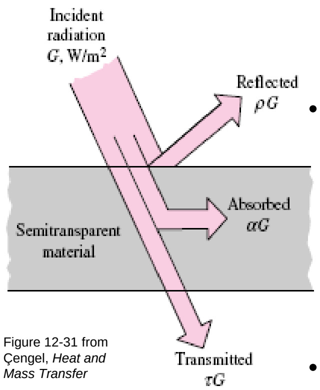


Figure 12-31 from Çengel, *Heat and Mass Transfer*

Properties

- When radiation, G, hits a surface a fraction ρG is reflected; another fraction, αG is absorbed, a third fraction τG is transmitted
- Energy balance: ρ + α + τ = 1

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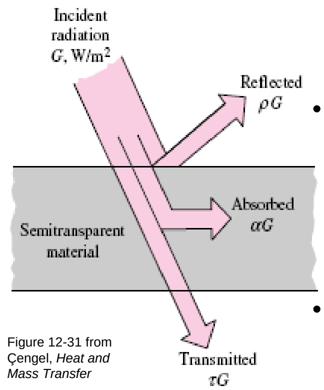

Figure 12-31 from Çengel, *Heat and Mass Transfer*

Properties II

- Fractions on previous chart are properties
  - Reflectivity, ρ
  - Absorptivity, α
  - Transmissivity, τ
- Energy balance: ρ + α + τ = 1

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Properties III

- As with emissivity, α, ρ, and τ may be defined on a spectral and directional basis
  - Can also take averages over wavelength, direction or both as with emissivity
  - Simplest case is no dependence on either wavelength or direction
  - Reflectivity may be diffuse or have angle of reflection equal angle of incidence

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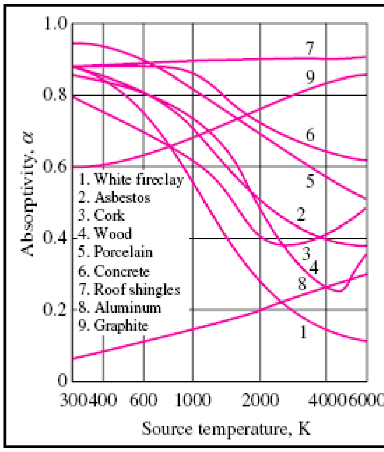

α Data

- Solar radiation has effective source temperature of about 5800 K

Figure 12-33 from Çengel, *Heat and Mass Transfer*

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Kirchoff's Law

- Absorptivity equals emissivity (at the same temperature)
- True only for values in a given direction and wavelength
- Assuming total hemispherical values of α and ε are the same simplifies radiation heat transfer calculations, but is not always a good assumption

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Effect of Temperature

- Emissivity, ε, depends on surface temperature
- Absorptivity, α, depends on source temperature (e.g. $T_{sun} \approx$ 5800 K)
- For surfaces exposed to solar radiation
  - high α and low ε will keep surface warm
  - low α and high ε will keep surface cool
  - Does not violate Kirchoff's law since source and surface temperatures differ

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TABLE 12–3

Comparison of the solar absorptivity α_s of some surfaces with their emissivity ε at room temperature

Surface	α_s	ε
Aluminum		
Polished	0.09	0.03
Anodized	0.14	0.84
Foil	0.15	0.05
Copper		
Polished	0.18	0.03
Tarnished	0.65	0.75
Stainless steel		
Polished	0.37	0.60
Dull	0.50	0.21
Plated metals		
Black nickel oxide	0.92	0.08
Black chrome	0.87	0.09
Concrete	0.60	0.88
White marble	0.46	0.95
Red brick	0.63	0.93
Asphalt	0.90	0.90
Black paint	0.97	0.97
White paint	0.14	0.93
Snow	0.28	0.97
Human skin (Caucasian)	0.62	0.97

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From Çengel, *Heat and Mass Transfer*

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